

Role of anharmonicities of nuclear vibrations in fusion reactions at subbarrier energies

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Abstract

We discuss the effects of double octupole and quadrupole phonon excitations in ^{144}Sm on fusion reactions between ^{16}O and ^{144}Sm at subbarrier energies. The effects of anharmonicities of the vibrational states are taken into account by using the *sdf*-interacting boson model. We compare the results with those in the harmonic limit to show that anharmonicities play an essential role in reproducing the experimental fusion barrier distribution. From the analysis of the high quality fusion data available for this system, we deduce negative static quadrupole moments for both the first 2^+ and 3^- states in ^{144}Sm . This is the first time that the sign of static quadrupole moments of phonon states in a spherical nucleus is determined from the data of subbarrier fusion reactions.

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Collective phonon excitations are common phenomena in fermionic many-body systems. In nuclei, low-lying surface oscillations with various multipolarities are typical examples. The harmonic vibrator provides a zeroth order description for these surface oscillations, dictating simple relations among the level energies and the electromagnetic transitions between them. For example, all the levels in a phonon multiplet are degenerate and the energy spacing between neighboring multiplets is a constant. In realistic nuclei, however, there are residual interactions which cause deviations from the harmonic limit, e.g., they split levels within a multiplet, change the energy spacings, and also modify the ratios between various electromagnetic transition strengths. There are many examples of two-phonon triplets ($0^+, 2^+, 4^+$) of quadrupole surface vibrations in even-even nuclei near closed shells. Though the center of mass of their excitation energies are approximately twice the energy of the first 2^+ state, they usually exhibit appreciable splitting within the multiplet. A theoretical analysis of the anharmonicities for the quadrupole vibrations was first performed by Brink *et al.* [1], where they related the excitation energies of three-phonon states to those of double-phonon triplets. For a long time, however, the sparse experimental data on three-phonon states had caused debates on the existence of multi-phonon states. The experimental situation has improved rapidly in recent years, and data on multi-phonon states are now available for several nuclei. As a consequence, study of multi-phonon states, and especially their anharmonic properties, is attracting much interest [2].

In many even-even nuclei near closed shells, a low-lying 3^- excitation is observed at a relatively low excitation energy, which competes with the quadrupole mode of excitation [3]. These excitations have been frequently interpreted as collective octupole vibrations arising from a coherent sum of one-particle one-hole excitations between single particle orbitals differing by three units of orbital angular momentum. This picture is supported by large E3 transition probabilities from the first 3^- state to the ground state, and suggests the possibility of multi-octupole-phonon excitations. In contrast to the quadrupole vibrations, however, so far there is little experimental evidence for double-octupole-phonon states. One reason for this is that E3 transitions from two-phonon states to a single-phonon state compete against E1 transitions. This makes it difficult to unambiguously identify the two-phonon quartet states ($0^+, 2^+, 4^+, 6^+$). Only in recent years, convincing evidences have been reported for double-octupole-phonon states in some nuclei, including ^{208}Pb [4] and ^{144}Sm [5].

Nuclear surface vibrations have also been studied in connection with nuclear reaction problems. For instance, the influence of nuclear surface vibrations on heavy-ion fusion reactions at energies below and near the Coulomb barrier has been investigated by many groups (see Ref. [6] for a review). These studies were later extended to include the effect of multi-phonon states [7, 8, 9]. It has been recognized by now as a general phenomenon that such channel couplings cause a significant enhancement of fusion cross sections relative to the predictions of one-dimensional barrier penetration models [6]. Recently, it was suggested that the effects of channel couplings can be visualized more effectively by studying the second derivative of the product of the fusion cross section and the center of mass energy with respect to the energy [10]. This quantity is conventionally called *fusion barrier distribution*, because it represents the distributed fusion barriers induced by the

coupling of the relative motion to nuclear intrinsic motion in the limit of degenerate spectrum, i.e. in the limit where the excitation energy of the nuclear intrinsic excitation is ignored. The excitation function of fusion cross sections has to be measured with very high accuracy at small energy intervals in order to deduce meaningful barrier distributions from the experimental data. Thanks to the recent developments in experimental techniques [11], such data are now available for several systems, and they have clearly demonstrated the sensitivity of the barrier distribution to the details of the channel coupling [12]. For example, the barrier distribution analysis of the recently measured accurate data on $^{58}\text{Ni} + ^{60}\text{Ni}$ fusion reaction has shown evidence for coupling of multi-phonon states in ^{58}Ni and ^{60}Ni [13]. The barrier distributions were shown to be quite sensitive to the number of phonons excited during fusion reactions. This suggests that subbarrier fusion reactions may provide an alternative method to identify the existence of multi-phonon states and to study their detailed properties such as anharmonicities.

The $^{16}\text{O} + ^{144}\text{Sm}$ fusion reaction, whose excitation function has recently been measured with high accuracy [12], could serve as a test case in this respect. It has been reported that inclusion of the double-phonon excitations of ^{144}Sm in coupled-channels calculations in the harmonic limit destroys the good agreement between the experimental fusion barrier distribution and the theoretical predictions obtained when only the single-phonon excitations are taken into account [14]. On the other hand, there are experimental [5, 15] as well as theoretical [16] support for the existence of the double-octupole-phonon states in ^{144}Sm . Reconciliation of these apparently contradictory facts may be possible if one includes the anharmonic effects, which are inherent in most multi-phonon spectra.

The aim of this Letter is to show that the anharmonicities indeed play an important role in the fusion reactions between ^{16}O and ^{144}Sm . We demonstrate that the anharmonic properties of the quadrupole and octupole vibrational excitations in ^{144}Sm strongly influence the shape of the fusion barrier distributions, and lead to a good agreement between the experimental data and theoretical predictions. The excitations of ^{16}O are not included as they are effectively incorporated in the choice of the bare potential [17]. We also estimate the magnitude as well as the sign of the quadrupole moments of the quadrupole and octupole single-phonon states of ^{144}Sm from the experimental fusion barrier distribution.

The *sdf*-interacting boson model (IBM) in the vibrational limit provides a convenient calculational framework to address these questions [18]. The vibrational limit of the IBM and the anharmonic vibrator (AHV) in the geometrical model are very similar, the only difference coming from the finite number of bosons in the former [19]. A model for subbarrier fusion reactions, which uses the IBM to describe effects of channel couplings, has been developed in Ref. [20]. Following [20], we assume that the Hamiltonian for the fusing system is given by

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + H_{IBM} + V_{coup}(\mathbf{r}, \xi), \quad (1)$$

where \mathbf{r} is the coordinate of the relative motion between the projectile and the target, μ is the reduced mass, and ξ represents the internal degrees of freedom of the target nucleus. H_{IBM} is the IBM Hamiltonian for the quadrupole and octupole vibrations in the target

nucleus, for which we assume the harmonic limit

$$H_{IBM} = \epsilon_d \hat{n}_d + \epsilon_f \hat{n}_f. \quad (2)$$

Here \hat{n}_d and \hat{n}_f are the number operators for d and f bosons, and, ϵ_d and ϵ_f are the excitation energies of the quadrupole and octupole vibrations, respectively. Note that we have neglected the two-body interactions in Eq. (2) that give rise to anharmonicities in the spectrum. The reason for this apparently self-defeating choice is that anharmonicities in level energies have only a marginal effect on the fusion excitation function and the barrier distribution. In fact, our studies show that the fusion barrier distribution does not depend so much on the excitation energies of the multi-phonon states once the energies of the single-phonon quadrupole and octupole states are fixed. As we will see later, the main effects of the anharmonicity on fusion barrier distributions come from the deviation of the transition probabilities from the harmonic limit.

The coupling between the relative motion and the intrinsic motion of the target nucleus is described by V_{coup} in Eq. (1), which consists of the Coulomb and nuclear parts. Following Ref. [20], and using the no-Coriolis approximation [7], they are given by

$$V_C(r, \xi) = \frac{Z_P Z_T e^2}{r} \left(1 + \frac{3}{5} \frac{R_T^2}{r^2} \frac{\beta_2 \hat{Q}_{20}}{\sqrt{4\pi N}} + \frac{3}{7} \frac{R_T^3}{r^3} \frac{\beta_3 \hat{Q}_{30}}{\sqrt{4\pi N}} \right),$$

$$V_N(r, \xi) = -V_0 \left[1 + \exp \left(\frac{1}{a} \left(r - R_0 - R_T (\beta_2 \hat{Q}_{20} + \beta_3 \hat{Q}_{30}) / \sqrt{4\pi N} \right) \right) \right]^{-1}. \quad (3)$$

Here, N is the boson number, the subscripts P and T refer to the projectile and target nuclei, respectively, and $R_0 = R_P + R_T$. The scaling of the coupling strength with \sqrt{N} is introduced to ensure the equivalence of the IBM and the geometric model results in the large N limit [20]. Further, β_2 and β_3 in Eq. (3) are the quadrupole and octupole deformation parameters, which are usually estimated from the electric transition probabilities using the expression $\beta_\lambda = 4\pi(B(E\lambda) \uparrow)^{1/2} / 3Z_T e R_T^\lambda$. However, this formula does not hold for anharmonic vibrators. Therefore, we treat β_2 and β_3 as free parameters and look for their optimal values to reproduce the experimental data. Finally, \hat{Q}_2 and \hat{Q}_3 in Eq. (3) are the quadrupole and the octupole operators in the IBM, which we take as

$$\hat{Q}_2 = s^\dagger \tilde{d} + s \tilde{d}^\dagger + \chi_2 (d^\dagger \tilde{d})^{(2)} + \chi_{2f} (f^\dagger \tilde{f})^{(2)},$$

$$\hat{Q}_3 = s f^\dagger + \chi_3 (\tilde{d} f^\dagger)^{(3)} + h.c., \quad (4)$$

where tilde is defined as $\tilde{b}_{l\mu} = (-)^{l+\mu} b_{l-\mu}$. When all the χ parameters in Eq. (4) are zero, quadrupole moments of all states vanish, and one obtains the harmonic limit in the large N limit. Non-zero values of χ generate quadrupole moments and are responsible for the anharmonicities in electric transitions.

Our coupled-channel calculations include a number of new features that improve on previous calculations. We do not employ the “constant coupling” approximation, which is often introduced in simplified calculations. Another important aspect of our formalism is that we do not introduce the usual linear coupling approximation by expanding the nuclear part in Eq. (3) with respect to the deformation parameters, but we keep the couplings

to the intrinsic motion to all orders. The full order treatment is crucial in order to quantitatively, as well as qualitatively, describe heavy-ion subbarrier fusion reactions [20, 21]. Also, we take into account the finite excitation energies in the target nucleus, which have been neglected in previous applications of the IBM to subbarrier fusion reactions [20]. Clearly, excitation energies of the order of 1 MeV, as typically encountered in vibrational nuclei, are too large to be ignored in fusion dynamics.

The model parameters are determined as follows. The standard prescription for boson number (i.e. counting pairs of nucleons above or below the nearest shell closure) would give $N = 6$. However, it is well known that the effective boson numbers are much smaller due to the $Z = 64$ subshell closure [19]. The suggested effective numbers in the literature vary between $N = 1$ and 3. We adopted $N = 2$ in our calculations, since there are experimental signatures for the two-phonon states, but no evidence for three-phonon states in ^{144}Sm . The parameters of the IBM Hamiltonian Eq. (2) are simply determined from the excitation energies of the first 2^+ and 3^- states in ^{144}Sm as $\epsilon_d = 1.66$ MeV and $\epsilon_f = 1.81$ MeV. The nuclear potential parameters are taken from the exhaustive study of this reaction in Ref. [14] as $V_0 = 105.1$ MeV, $R_0 = 8.54$ fm and $a = 0.75$ fm. Finally, the target radius is taken to be $R_T = 5.56$ fm.

The results of the coupled-channels calculations are compared with the experimental data in Fig. 1. The upper and the lower panels in Fig. 1 show the excitation function of the fusion cross section and the fusion barrier distributions, respectively. The experimental data are taken from Ref. [12]. The dotted line is the result in the harmonic limit, where couplings to the quadrupole and octupole vibrations in ^{144}Sm are truncated at the single-phonon levels. The deformation parameters are estimated to be $\beta_2=0.11$ and $\beta_3=0.21$ from the electric transition probabilities. The dotted line reproduces the experimental data of both the fusion cross section and the fusion barrier distribution reasonably well, though the peak position of the fusion barrier distribution around $E_{cm} = 65$ MeV is slightly shifted. As was shown in Ref. [14], the shape of the fusion barrier distribution becomes inconsistent with the experimental data when the double-phonon channels are included in the harmonic limit (the dashed line). The good agreement is recovered when one takes the effects of anharmonicity of the vibrational motion into account. These results are shown in Fig. 1 by the solid line. This calculation has been performed using the parameters, $\beta_2 = 0.13$, $\beta_3 = 0.23$, $\chi_2 = -3.30$, $\chi_{2f} = -2.48$, and $\chi_3=2.87$, which are obtained from a χ^2 fit to the fusion cross sections. The χ^2 fit gave a unique result, regardless of the starting values. The non-zero χ values indicate the anharmonic effects in the transition operators. The slight change in the values of the deformation parameters from those in the harmonic limit results from the renormalization effects due to the extra terms in the operators given in Eq. (4). Note that the solid line agrees with the experimental data much better than the dotted line.

One of the pronounced features of an anharmonic vibrator is that the excited states have non-zero quadrupole moments [3]. Using the χ parameters extracted from the analysis of fusion data in the E2 operator, $T(\text{E}2) = e_B \hat{Q}_2$, we can estimate the static quadrupole moments of various states in ^{144}Sm . Here, e_B is the effective charge, which is determined from the experimental $B(\text{E}2; 0 \rightarrow 2_1^+)$ value as $e_B = 0.16$ *eb*. For the quadrupole moment of the first 2^+ and 3^- states, we obtain -0.28 b and -0.70 b, respectively. The nega-

tive sign of the quadrupole moment of the octupole-phonon state is consistent with that suggested from the neutron pick-up reactions on ^{145}Sm [22].

In the case of rotational coupling, fusion barrier distributions strongly depend on the sign of the quadrupole deformation parameter through the reorientation term. Also, it has been reported that fusion barrier distributions are very sensitive to the sign of the hexadecapole deformation parameter [23]. Similarly, it is likely that the shape of fusion barrier distributions changes significantly when one inverts the sign of the quadrupole moment in a spherical target. Fig. 2 shows the influence of the sign of the quadrupole moment of the excited states on the fusion cross section and the fusion barrier distribution. The solid line is the same as in Fig. 1 and corresponds to the optimal choice for the signs of the quadrupole moments of the first 2^+ and 3^- states. The dotted and dashed lines are obtained by changing the sign of the χ_2 and χ_{2f} parameters in Eq. (4), respectively, while the dot-dashed line is the result where the sign of both χ_2 and χ_{2f} parameters are inverted. The change of sign of χ_2 and χ_{2f} is equivalent to taking the opposite sign for the quadrupole moment of the excited states. Fig. 2 demonstrates that subbarrier fusion reactions are indeed sensitive to the sign of the quadrupole moment of excited states. The experimental data are reproduced only when the correct sign of the quadrupole moment are used in the coupled-channels calculations. Notice that the fusion excitation function is completely insensitive to the sign of the quadrupole moment of the first 2^+ state, but strongly depends on that of the first 3^- state. In contrast, the fusion barrier distribution can probe the signs of the quadrupole moments of both the first 2^+ and 3^- states. This study shows that the sign of quadrupole moments in spherical nuclei can be determined from subbarrier fusion reactions, especially through the barrier distribution.

In summary, we have analyzed the experimental fusion excitation function for $^{16}\text{O} + ^{144}\text{Sm}$ reaction with a model which explicitly takes into account the effects of anharmonicity of the vibrational modes of excitation in ^{144}Sm . We have focused on the anharmonic effects of the phonon excitations in ^{144}Sm and found that the best fit to the experimental data requires negative quadrupole moments for the first 2^+ and the first 3^- states. As a general conclusion, we find that heavy-ion subbarrier fusion reactions, and in particular, barrier distributions extracted from the fusion data, are very sensitive to the sign of the quadrupole moments of phonon states in the target nucleus.

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Figure Captions

Fig. 1: Comparison of the experimental fusion cross section (the upper panel) and fusion barrier distribution (the lower panel) with the coupled-channels calculations for $^{16}\text{O} + ^{144}\text{Sm}$ reaction. The experimental data are taken from Ref. [12]. The solid line shows the results of the present IBM model including the double-phonon states and anharmonic effects. The dotted and the dashed lines are the results of the single- and the double-phonon couplings in the harmonic limit, respectively.

Fig. 2: Dependence of the fusion cross section and barrier distribution on the sign of the quadrupole moment of the excited states in ^{144}Sm . The meaning of each line is indicated in the inset.



